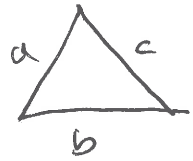
EJERCICIO 2

a/ $P(x)=x^3-16x^2+81x-128=0$, a, b, c las raíces de P(x)



$$A=\sqrt{s\cdot(s-a)(s-b)(s-c)}$$

FÓRMULA DE HERÓN

$$s=\frac{a+b+c}{2}$$ (Semiperímetro)

$$P(x)=(x-a)\cdot(x-b)\cdot(x-c)=x^3-(a+b+c)x^2+(ab+ac+bc)x-abc$$

$$=x^3-16x^2+81x-128$$

Igualando coeficientes tenemos las ecuaciones de Cardano-Vieta:

$$a+b+c=16 \longrightarrow s=\frac{a+b+c}{2}=8$$

$$ab+ac+bc=81$$

$$abc=128$$

$$(s-a)(s-b)\cdot(s-c)=(8-a)\cdot(8-b)\cdot(8-c)=P(8)$$

$$P(8)=8^3-16\cdot 8^2+81\cdot 8-128=8$$

Luego aplicando la fórmula de Herón

$$A=\sqrt{\underbrace{s}_{8}\,\underbrace{(s-a)(s-b)(s-c)}_{8}}=\sqrt{8\cdot 8}=\boxed{8\ u^2}$$

b) $\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx = \left[\begin{array}{l} x=t^4,\ t=\sqrt[4]{x} \\ \sqrt{x}=t^2 \\ dx=4t^3dt \end{array}\right] = \int \frac{\sqrt[3]{1+t}}{\cancel{t^2}} \cdot 4t^{\cancel{3}}dt =$

$= 4\int \sqrt[3]{1+t} \cdot t\,dt = \left[\begin{array}{l} 1+t=z^3,\ z=\sqrt[3]{1+t} \\ dt=3z^2dz \\ t=z^3-1 \end{array}\right] = 4\int z\cdot(z^3-1)\cdot 3z^2dz$

$= 12\int (z^6 - z^3)\,dz = 12\frac{z^7}{7} - 3z^4 + C = \frac{12}{7}\left(\sqrt[3]{1+\sqrt[4]{x}}\right)^7 -$

$z=\sqrt[3]{1+t}$

$z=\sqrt[3]{1+\sqrt[4]{x}}$

$- 3\cdot\left(\sqrt[3]{1+\sqrt[4]{x}}\right)^4 + C$

$$\boxed{I = \frac{12}{7}\left(\sqrt[3]{1+\sqrt[4]{x}}\right)^7 - 3\left(\sqrt[3]{1+\sqrt[4]{x}}\right)^4 + C \qquad C\in\mathbb{R}}$$